SOLVING GEOMETRICAL LOCUS PROBLEMS USING DYNAMIC INTERACTIVE GEOMETRY APPLICATIONS

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1. Introduction

A geometrical locus is mathematically defined as the set of all points or lines that satisfy or are determined by *specific conditions*. Even this definition seems to be a clear one, geometrical problems which proposed the searching of a locus have proved to be not so simple for be solved by pupils [1]. Practically, most of pupils have difficulties when demonstrating a geometrical locus. To tackle these kinds of problems when teaching, it can be started to the idea that all the geometrical locus problems are integrated in the family of *problems with equivalent conditions*. More than that, the geometrical locus problems look like the problems (theorems) which reciprocal are true. In this sense, the pupils have to know clearly the theorem structure, the way of its formulation and the fact that a reciprocal proposition becomes a reciprocal theorem if this is demonstrated as true. In essence, the geometrical problems where the locus is known are problems in which a multitude of plan points is defined in two ways, for the pupils remaining the task to demonstrate that those two multitudes are equal. That's why, from the methodological point of view, the teacher's responsibility has to be focused on the pupils knowledge concerning the definition of two equal multitudes and the theorem equivalent with that definition [2].

As the practice proved that the geometrical locus problems are difficult to be understood and solved by pupils, the traditional didactic method which involved the using of the blackboard and chalk became over fulfilled. New didactical strategies and methods have to be taken into consideration for presenting the geometrical locus problems and, ICT has to be integrated as a main component. One of the best opportunity for presenting ICT as a specific element of a didactical strategy appeared together with the approval of the *VccSSe* Project.

2. The *VccSSe* Project

The three years Socrates Comenius 2.1 European Project "*VccSSe - Virtual Community Collaborating Space for Science Education*" (http://vccsse.ssai.valahia.ro) has started in October 2006 and proposed - as declared objective - on targeting to adapt, develop, test, implement and disseminate training modules, teaching methodologies and pedagogical strategies based on the use of *Virtual Instruments*, with the view to their implementation in the classroom, through ICT tools. The main objective of the project has been achieved taking into account the specific particularities of different countries involved in the partnership. The project is coordinated by *Valahia University of Târgovişte, Romania*, the partnership being composed by 9 educational institutions from Romania, Spain, Poland, Finland and Greece.

One of the most important outcomes of the project was to create and develop specific materials for training on using *Virtual Instrumentation in Science Education* [3]. The training materials were dedicated to in-service teachers from primary and secondary schools involved in Sciences subjects in the partners' countries. Finally, the training materials presented four selected *Virtual Instrumentation environments* (*LabView*, *Crocodile Clips*, *Cabri Geometry* and *GeoGebra*) [4] and the participants - having also in view their background - were required to choose one of the software environments for understanding its main functions and creating at least one learning object that has to include a *VI* application.

The *Training Modules* were provided using the *Moodle* (*Modular Object-Oriented Dynamic Learning Environment*) e-learning platform [5], [6]. *VccSSe* training modules tutors and participants experienced the *Moodle*'s course organisation and features and expressed their opinions in specific discussions.

3. Cabri Geometry II Plus Software

Cabri Geometry II Plus allows the dynamically exploration of Euclidian, transformational and coordinate geometry. It makes the Mathematics concepts easier to learn thanks to its *kinaesthetic* learning approach. The geometric figures, equations or graph functions are easy to be created on the *Cabri* screen, practically becoming manipulable objects. In this way, the software gives to pupils the tools and motivation to dig deeper and actively explore and also to be creative contrasting the traditional inactive methods and those based on using the pencil and the paper [7]. As a geometrical shape can be simply modified through translation, the user can visualize infinity of geometrical constructions on the screen, with the same characteristics but with different shapes. Each geometrical shape produced with *Cabri Geometry II Plus* represents in fact a class of geometrical constructions with common geometrical properties. The variety of the offered instruments gives to the users the possibility to select the most proper one in order to structure a strategy for solving a problem and build a strategy for obtaining the results in many ways. At the same time, *Cabri* offers a set of instruments necessary for student's self-evaluation and self-control. Finally, *Cabri* sustains a development in the interaction between the visual and conceptual elements of the geometrical logics. Students can create and verify their hypothesis, create alternatives for the geometrical construction or can submit the images into document processors and send them on the Internet via *Cabri Java* [8], [9].

4. Examples of Solving Geometrical Locus Problems with Cabri Geometry II

The *Training Modules* developed in the frame of the *VccSSe* Project allowed the participants to choose the topic and prepare the related learning objects. Based on the specific curricula for primary and secondary schools, the Mathematics teachers chose various topics, the geometrical locus problems becoming a provocative challenge. In this respect, the teachers felt that when using *Cabri*, ones of the most interesting applications for pupils are dedicated for solving geometrical locus problems. After six weeks of work, the learning objects were ready and uploaded in the *Moodle* platform. In the following examples, two designed virtual experiments which involve problems related to geometrical locus are presented.

• Example 1 - The mean line of a triangle as a geometrical locus

Let the triangle *NOI* fixed and *A*, a mobile point on the line [*OI*]. Find the geometrical locus of *U*, the middle of the segment [*NA*].

Figure 1 presents the construction created for the problem exposed above. The graphical representation of the problem was a real help for the students who discovered the geometrical locus by searching the particular cases of *A* as a variable point - if *A* = *O*, then *U* = *L*, the middle point of [*NO*]. If *A* = *I*, then *U* = *C*, the middle point of [*NI*]. Last, if *A* is between *O* and *I*, then *U* is situated on (*LC*).

Using *Cabri Geometry*, it can be added also the effect of animation to point *A* (*Animation* key - click on *A*) and it can be noticed the next fact: if *A* crosses [*OI*] then *U* crosses the mean line [*LC*]. This line can be enlightened following the option *Show Button* (*Hide / Show Button* key).

• Example 2 - The circle as a geometrical locus in a parallelogram

Let the parallelogram *ABCD* with the *A* and *B* points as fixed ones and *C* and *D* points as mobile ones, so that the line *DC* becomes parallel with *AB*. Find the geometrical locus point *X* - the projection of the *A* point on the line limited by points *D* and *M* (the middle point of [*BC*]).

The geometrical locus is represented by the circle which has the centre in the *B* point and *AB* as circle ray (where the points *A* and *E* - diametrically opposite - are excluded). Figure 2 illustrates the geometrical constructions obtained using *Cabri Geometry II* software. The second graphical representation illustrates the situation in which the solver can follow the moving of *X* point on a circle with *B* point as centre and *BX* = *AB* as ray. An important advantage is offered by the using of colours for drawing the figures to fill inside the geometrical construction. The colours are recommended to be used in order to draw attention on the essential elements of the problem and to ease the understanding and memorizing of the basic principles of the problem.

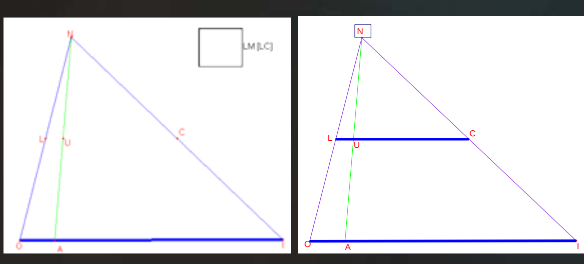

Fig. 1 The Cabri Geometry II construction created for Example 1
left - Initial representation; right - Animation effect added

The presented experiments, together with others realized by the participants in the frame of the mentioned *Training Modules*, were uploaded in the *VccSSe Products Matrix* - an on-line database which includes learning objects / virtual experiments developed for Science education. The *Matrix* is open for public access and downloading for the educational community with the view to promote the using of the virtual experiments for educational purposes. The e-learning technologies can follow the same philosophy of working through an e-learning platform and developing special web-spaces for presenting good practices or other results as examples (like the case of *VccSSe* Project).

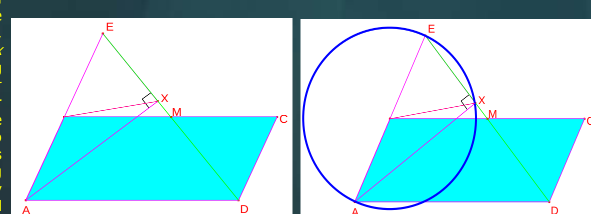

Fig. 2 The Cabri Geometry II construction created for Example 2
left - Initial representation; right - Animation effect added

5. Conclusion

Including in the Science lessons dedicated software for creating virtual experiments can have a huge efficiency on the didactic strategies aimed to teaching and learning. It was easy to conclude that in the case of geometrical locus problems, the methodological aspects of teaching were improved once the ICT was considered as a main component. The didactic activities emphasized - on the one hand - the importance of the virtual interaction between the learner and *Cabri Geometry II Plus* software. On the other hand, it is important to mention that the presented activities were designed in a specific way for encouraging the learners to take an investigative perspective, make self-corrections, formulate and verify specific conjectures. *Science in-service teachers* became learners for six weeks, worked on the *Moodle* platform and also created, designed and polished virtual experiments with a view to their implementation as learning objects in the classrooms. The virtual instrumentation offered to the teachers the possibility to develop viable didactic strategies in which are possible to combine classical techniques of Science teaching with new elements that ICT. More than that, the teachers became accustomed to the principles of e-learning and tried to fulfill the tasks and respect the stipulated deadlines.

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