AN ACTIVITY-BASED VIRTUAL SPACE FOR THE LEARNING OF GEOMETRICAL CONCEPTS USING DYNAMIC GEOMETRY SYSTEMS

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ABSTRACT

This paper presents the design of a virtual space for Mathematics teacher education across Europe for the appropriate use of Information and Communication Technologies (ICT) in their teaching practices. The design of this virtual space is based on social and constructivist theories of learning. The well-known educational software Cabri-Geometry II (Laborde, 1990) was selected to help teachers introduce Dynamic Geometry Systems into their teaching practices. The basic structural element of the aforementioned virtual space is the learning activity using the tools provided by Cabri. This space is integrated into a wider virtual space named 'Virtual Community Collaborating Space for Science Education (VccSSe) which is dedicated for Sciences teacher education across Europe, aiming at the adaptation, development, testing, implementation and dissemination of training modules, teaching methodologies and pedagogical strategies based on the use of Virtual tools and Instruments, with the view of implementing them in the classroom, through the use of Information and Communication Technology tools.

INTRODUCTION

It is widely accepted that Information and Communication Technologies (ICT) have influenced our entire social and economic life as well as the teaching and learning of all subjects (Solloway, 1993; Noss and Hoyles, 1996). In particular, the use of appropriately designed educational software can catalytically affect the changes in the whole learning context in terms of learning content and the roles of both teachers and learners involved in the teaching and learning process (Noss, 1988; Noss and Hoyles, 1992). In fact, in the context of ICT, modern social and constructivist perspectives of teaching and learning can be realized (Papert, 1980; Balacheff and Kaput, 1996; Kordaki, 2003a). The use of the Internet as a means of communication but also as a mediator in both teaching and learning also opens new vistas for the realization of social and constructivist learning practices (Miranda, Pinto, 1996; Briano, Midoro, and Trentin, 1977). In the Internet context, learners from different geographic places around the world can be actively involved in their learning and also communicate their knowledge through numerous many-to-many communications at any time and in any place. From this perspective, the need for training primary and secondary level education teachers in the use of ICT in education is of vital importance not only for their integration into the modern social and educational context created, but also for the integration of ICT into education (European Commission, 1997). The necessity of training teachers in ICT concerns the acquisition of basic technical and pedagogical skills related to the use of ICT so that they will be capable of integrating it into the teaching and learning of the subject-matter they teach (Davis and Tearle, 1998). The possibility of training teachers in their place and time via the Internet is essential for them, because they are workers in different geographic regions, and consequently it is not easy for them to be available for training in specific places and time, as needed in conventional forms of training (Michaelides, 2000). It is also essential to provide in-service teachers with opportunities to interact, cooperate and communicate within a wide international virtual space, in order to learn and exchange ideas about the use of ICT in the teaching and learning of specific learning subjects. It is worth noting that, until now, there are few virtual cooperative spaces for training Sciences (Informatics, Biology, Chemistry, Maths and Science) in Europe and all of them are addresses of University teachers and students (mainly in Engineering studies). In the context described above, the development of a Virtual Community Collaborating Space for Science Education (VccSSe) has been accepted as a European project within the COMENIUS-SOCRATES framework. The project is aimed to adapt, develop, test, implement and disseminate training modules, teaching methodologies and pedagogical strategies based on the use of Virtual tools and Instruments, with the view of implementing them in the classroom, through the use of Information and Communication Technology tools.

The proposed Virtual Space will allow every teacher to find virtual tools for his / her own needs and develop proper pedagogical approaches to use them. Most of the pedagogical and didactical approaches proposed give the Virtual Space a strong innovative character. Working collaboratively inside the Space could be a bridge between teachers, helping them to feel part of a larger community of practice - a learning team - through changing their ideas and experiences. It may also support recognition of Europe wide similarities and problematic areas in science education and the development of pedagogical solutions for virtual education. In the context of VccSSe, a virtual space is to be formed for the training of mathematics teachers in the use of ICT in their practices (VccSSe-Math). This space has been designed to support teachers in primary and secondary education to exploit the advantages of

Dynamic Geometry Systems in their teaching practices. The basic structural element of the design of VccSSe-Math space is the learning activity. Such e-learning space has not yet been reported.
In the following part of this paper, the rationale of the design of VccSSe-Math space is presented, followed by a demonstration of the specific design of this space and, finally, summarizing remarks and proposals for future work are made.

THE RATIONALE OF VccSSe-MATH SPACE

Cornerstones for the creation of an open learning environment via the Internet are: a) the teaching and learning approaches that it supports, b) the technology that is used for the realization of teaching and learning, and c) the particular learning community in which the environment for learning via this process is addressed (Pulkkinen and Ruotsalainen, 1998).

Teaching and learning approaches: Constructivist considerations regarding knowledge construction acknowledge that learning is an active, subjective and constructive activity (von Glasersfeld, 1995). Moreover, social approaches to learning recognize the importance of communication (Vygotsky, 1978) and of tools (Cobb, 1997), and more specifically the role of computers and the Internet as means of mediation in the process of knowledge construction by individuals (Noss and Hoyles, 1996; Briano, Midoro and Trentin, 1977). Learning activities also play a significant and central role in the entire learning process (Leont'ev, 1981, as cited by Noss and Hoyles, 1996; Nardi, 1996). In fact, appropriately designed learning activities can play a crucial role in motivating learners to be actively involved in their learning. Constructivist design of learning activities emphasizes the fundamental concepts of the learning subject in focus and not its details (von Glasersfeld, 1995; Vygotsky, 1978; Nardi, 1996). Consequently, emphasis is placed on the necessary activities to be performed by students for their effective learning of the subject matter. Moreover, holistic learning activities can help learners to acquire a global view of the learning subject in focus. Problem-solving activities that put learners into an investigative mode can encourage them to construct their knowledge actively as well as to acquire certain essential problem-solving skills. In this way, learners can be encouraged to form and verify conjectures, hypotheses and formulas as well as to develop self-correction strategies. In addition, it is essential that student difficulties in the understanding of the concepts in focus be taken into account. Finally, the kind of activities that can help learners express their inter-individual and intra-individual varieties is of major interest. To this end, multiple-solution activities to be performed in contexts providing a variety of tools are the most appropriate. Students can select the tools that suit them in order to express their knowledge by constructing an individual solution to the given tasks. Thus, students can be encouraged to express their inter-individual differences. Furthermore, each individual student can express a different kind of knowledge regarding the learning subject in focus. As a result, they can express their intra-individual variety and acquire different learning styles.

Use of suitable technology. Constructivist computer learning environments can act as catalysts in the whole learning context and can play an essential, crucial and exceptional role in student learning (Hillel, 1993; Noss & Hoyles, 1996). These environments can also act as scaffolding factors for the development of mathematical activity, while also helping students to reorganize their thinking (Hillel, 1993) and to make connections between informal and typical mathematics as well as to experiment and make generalizations exploiting their experience (Noss, 1988). In particular, computer learning environments providing a variety of tools of different cognitive transparency could encourage students to select those most appropriate to express their knowledge. These different tools can provide students with opportunities to study the concepts in focus in multiple representation systems in order to express both inter-individual and intra-individual varieties (Janvier, 1987). Most learner difficulties are found in the gap between their intuitive knowledge and the knowledge they need to express in the proposed representation systems (RS) to be used (Janvier, 1987). For example, prepositional, symbolic and abstract representation systems prevent some learners (usually beginners) from expressing their knowledge, the same systems being intended for use by advanced learners. Contrariwise, metaphors of everyday life and visual RS are more suitable for beginners. Multiple RS have been proposed to enable individual learner variety as well as to enable each learner to acquire a broader view of the subject to be learned (Kordaki, 2003b).

Cabri Geometry II (Laborde, 1990) is widely-known educational software designed to support constructivist mathematical learning settings. In particular, Cabri provides students with potential opportunities in terms of: a) *means of construction*, providing a rich set of tools to perform a variety of geometrical constructions referring to a variety of concepts concerning Euclidean Geometry. These tools can be exploited by students to perform a number of different geometrical constructions and to deal with a variety of geometrical problems. *b) Tools to construct a variety of representations, both numerical and visual*, such as geometrical figures, tables, equations, graphs and calculations. These representations are of different cognitive transparency; consequently, students can select the most appropriate tools to express their knowledge. In this way, students have the possibility of expressing both inter-individual and intra-individual differences. The representation systems used also affect the kind of knowledge that students construct (Kordaki, 2003b; Mariotti, 1995) c) *Linking representations*, by exploiting the interconnection of the different representation modes provided. d) *Dynamic, direct manipulation of geometrical constructions* by using the 'drag mode' operation. This operation gives learners the possibility of experimenting with geometrical constructions and forming and verifying hypotheses and conjectures by handling, in a physical sense, the theoretical objects which appear as diagrams on the computer screen (Laborde and

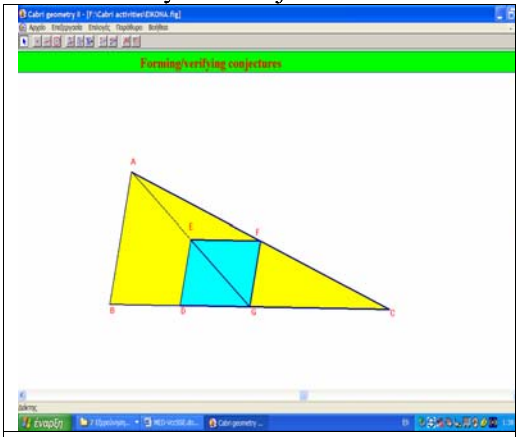
Laborde, 1995). In these Cabri-constructions, their geometrical properties are retained under dragging, while their visual output differs. The 'drag mode' can be used in three modes: as an 'exploratory' mode, a 'verification' mode and an 'adjustment' mode (Kordaki and Balomenou, 2006). By using 'drag mode', students can also form dynamic views of the concepts in focus (Mariotti, 1995). e) *The possibility of collecting large amounts of numerical data*. Cabri provides the opportunity for automatic tabulation of large amounts of specific numerical data viewed as appropriate for the study of the geometrical concept in focus. In particular, the 'drag mode' can be used in combination with automatic measurements of specific elements of the geometrical constructions under study. These measurements can be automatically tabulated, providing learners with opportunities to reflect on them and form and verify conjectures about specific geometrical concepts and relationships. f) *Interactivity and feedback*; intrinsic visual feedback and extrinsic numerical feedback, providing learners with opportunities to form and verify conjectures as well as to self-correct their constructions. This is important, as learner actions are closely connected with their consequences, contrary to the static and silent paper and pencil environment where there is no possibility of providing immediate response to those actions (Kaput, 1994), g) *Presenting information to the students in text form*, for example, the presentation of the tasks at hand, *h) Capturing the history of student actions* to provide teachers and researchers with a valuable amount of data for further studies, and i*) Extension*. Certain operations could be added as buttons on the Cabri interface following the formation of specific macros.

Cabri presents strong capabilities for the design of learning activities that encourage learners to: take an investigative perspective, express their inter-individual and intra-individual learning differences, make self-corrections, formulate and verify conjectures and exploit the advantages from the negotiation of their knowledge with the knowledge of their classmates in cooperative settings (Straesser, 2001; Kordaki & Balomenou, 2006). In addition, authentic meaningful real life learning activities can be integrated within the context of Cabri, activities that can develop strong learner motivation.

THE STRUCTURE OF VccSSe-MATH SPACE

The basic structure of VccSSe-math space is the learning activity. Consequently, six types of learning activities (Laborde, 2001; Kordaki and Balomenou, 2006; Kordaki, 2005) have been designed to be performed by the teachers using the tools of Cabri-Geometry II during their participation in the context of VccSSe-math space, namely:

a) *Forming/verifying conjectures by focusing on the alteration of a geometrical construction using the drag-mode operation.* For example, when a student draws a triangle ABC, its median AG and D, E, F the midpoints of BG, AG and AC respectively, they can conjecture that ‘the quadrilateral DEFG is a parallelogram’ and also verify this conjecture formed somehow during their experience.



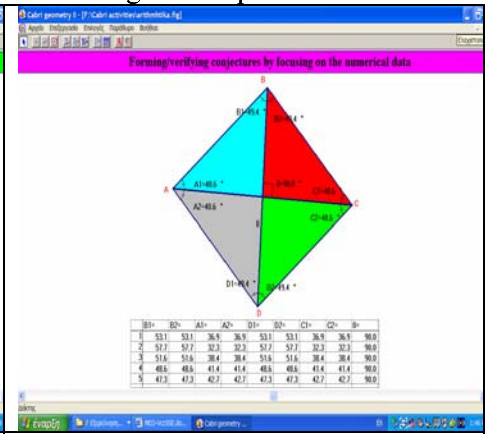


Figure 1a	Figure 1b

Figure 1. Examples of (a) and (b) types of activities consisting of the proposed e-learning course

b) *Forming/verifying conjectures by focusing on the numerical data automatically collected during the alteration of a geometrical construction using the drag-mode operation.* A case in point is when a student draws a rhombus ABCD and its diagonals AC and BD as well as their intersection point O, then automatically measures the angles ABD, DBC, BCA, ACD, CDB, BDA and BOC, and tabulates the data automatically produced by the aforementioned measurement operations. By focusing on these numerical data, this student can conjecture that ‘The diagonals of a rhombus are perpendicular and bisect its angles’. In this way, students can also verify conjectures formed somehow (eg. using their visual perception) during their experience.

c) Verifying a formula by focusing on the numerical data automatically collected during the alteration of a geometrical construction using the drag-mode operation. For instance, students can verify the generalization of the Pythagorean theorem: The square of a side of a triangle found opposite the acute (obtuse) angle is equal to the sum of the squares of the 2 other sides, decreased (increased) by double product of one of them multiplied by the projection of the other one onto this (see Figure 2a).

d) *Multiple-solution activities.* As Cabri provides a variety of tools and operations, these can be effectively combined to support the performance of multiple-solution activities. In the context of these activities, it is possible to integrate all the possibilities provided by Cabri into any of the possible different types of activities. For example, students could be asked to form pairs of similar triangles in any possible way using the variety of tools provided. Some of the possible solutions one can see in Figure 2b.

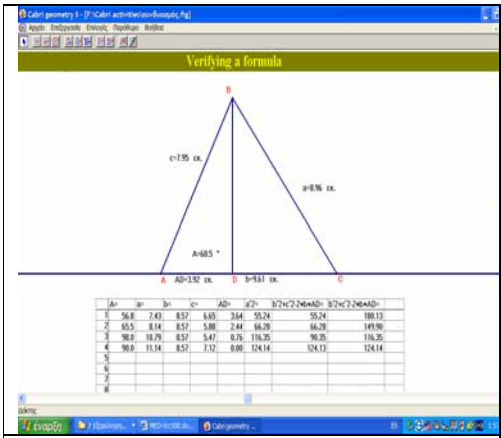


Figure 2a

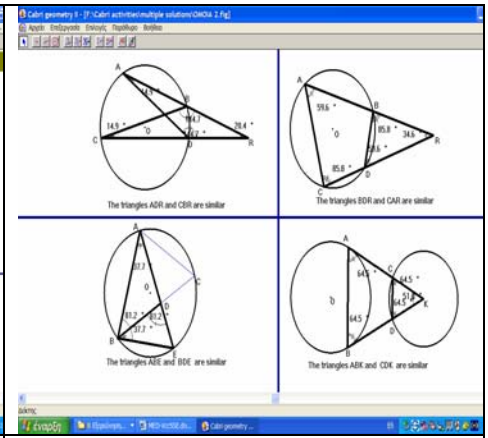


Figure 2b

Figure 1. Examples of (c) and (d) types of activities consisting of the proposed e-learning course

e) *Black-box activities.* Students can participate in activities where they have to explore geometrical constructions with some of their properties hidden, which they then have to discover. To illustrate this, students can be asked to investigate the specific properties of the triangles illustrated on the computer screen (see Figure 3a). It is worth noting that one pair of these triangles is arbitrary while the other pair has been constructed as similar triangles. Students can reach this conclusion by manipulating these pairs of triangles using the 'drag mode' operation.

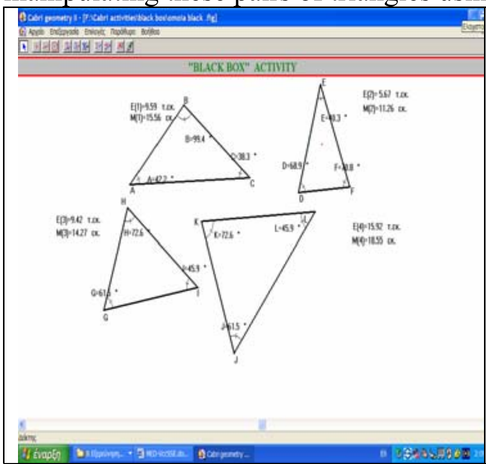


Figure 3a

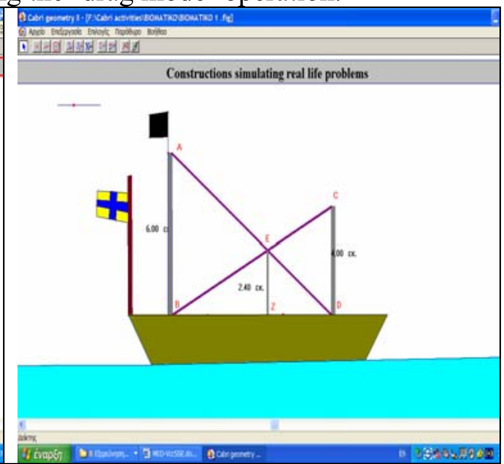


Figure 3b

Figure 1. Examples of (e) and (f) types of activities consisting of the proposed e-learning course

f) *Constructions simulating real life problems*. Such real life problems can help students to develop strong motivation in their learning and approach mathematics

as a human activity (Bishop, 1988) as well as put mathematical concepts into an interdisciplinary context (Clements, 1989). As an example, let's consider the following problem: Two masts of a boat are 4 and 6 metres high respectively. Cables connect the top of the first with the base of the other. The cables intersect at point E, which is 2.4 metres above the deck. How can the captain, moving the masts, add a third, 3 metre cable, to the intersection, so as to achieve better support of the sails? (see Figure 3b).
Finally, it is worth mentioning that teachers are provided with technical instructions about the use of Cabri- tools. In addition, teachers are provided with information related to theoretical issues about modern social and constructivist learning theories in the context of Cabri-Geometry II.

SUMMARIZING REMARKS AND FUTURE PLANS

This study presented the design of a virtual cooperative space (the VccSSe-Math space) dedicated for Mathematics teacher education across Europe with regard to the integration of ICT in their teaching practices. The well known educational software Cabri-Geometry II was selected to provide these teachers with the appropriate tools to construct learning activities for a variety of concepts of Euclidean Geometry. In particular, the aim of this space is to encourage the Mathematics teachers across Europe to learn how to use and exploit the tools provided by Cabri-Geometry II in order to design constructivist learning activities for their students. The design of VccSSe-Math space was based on social and constructivist theories of learning. The basic structural element of this space is the learning activity. In fact, the content of each thematic area included in VccSSe-Math space is organized in terms of the following six types of learning activities: a) Forming/verifying conjectures by focusing on the alteration of a geometrical construction using the drag-mode operation. b) Forming/verifying conjectures by focusing on the numerical data automatically collected during the alteration of a geometrical construction using the drag-mode operation. c) Verifying a formula by focusing on the numerical data automatically collected during the alteration of a geometrical construction using the drag-mode operation. d) Multiple-solution activities. e) Black-box activities. f) Constructions simulating real life problems. Our plans for the future include the creation of a community of mathematics teachers across Europe in order to enable them to use the tools provided by Cabri to design and implement in their teaching practices learning activities taking into account modern social and constructivist learning theories at the same time exploiting the advantages of the tools provided.

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