THE ROLE OF VIRTUAL EXPERIMENTS IN TEACHING MATHEMATICS

A. PIETRZYK[*,1]

[1] Gimnazjum im. Powstańców Śląskich w Miedźnej, Poland

Abstract:
The aim of this article is to show the usage of GeoGebra. These are not theoretical reflections. All of the presented didactic situations happened in my lessons. I have conducted lessons in various classes. I have done each of the experiments together with my students. It is not, therefore, a presentation of the possibilities of the software. It is rather a presentation of practical use of it in discovering theorems, stating thesis on the basis of the conducted experiments, confirming implications searching for counter examples as well as combating hypotheses. I also present situations, which had not been planned but happened of their own accord as a result of students using GeoGebra. I also touch upon a certain hazard that might occur while using GeoGebra.

Keywords: GeoGebra, virtual experiment, modeling, stating hypotheses, verifying hypotheses

1. Introduction

The common opinion shared both by students and their parents is that mathematics is the most difficult school subject. If asked which school subject they like the least or they fear the most, students will often point at mathematics. Nevertheless, on the basis of my experience I have arrived at the conclusion that there are sections of mathematics that students just adore, show vivid interest in or get deeply involved in. Moreover, they get better results in other sections. There was such a situation in my class during the lessons on basics of the probability theory. During those lessons, the students conducted a number of random experiments and on their basis they formed conclusions. They played random games and strategic-random games. They estimated the players' chances and were looking for strategies that would bring about victory. They estimated average profits of the games. They matched the fees to the prizes. Finally, they developed a game themselves. They analysed its profits and, in final, they play the game, confirming its profitability. The results were beyond my expectations. The students showed a great deal of ingenuity and activity. First of all, they used their skills and experience in practice. During the reflection on this

[*] Corresponding author: e-mail: a.p3k@interia.pl

phenomenon (I mean my lessons), I arrived at the conclusion that the results had been brought abort by the following:

1) students were carrying out experiments and, therefore, the lessons were very dynamic;
2) the students used their skills to solve real/actual, and not theoretical, problems;
3) on the basis of the experiments students form conclusions and hypotheses;
4) theoretical calculations were confirmed by experiments;
5) experiments evoke students' curiosity and encourage them to ask questions.

As it can be easily perceived, the success of the classes can be attributed, to a great extent, to experiments. I decided to try out this way of teaching during other classes. Such an opportunity arose after the course "**Virtual Instrumentation in teaching science subjects"**. The software called GeoGebra which I learnt about there turned out to be an excellent tool to prepare not only virtual experiments but also interesting interactive exercises. I prepared a few lessons based solely on GeoGebra as well as a number of exercises and experiments as an assisting part to a lesson.

2. How to prepare a good lesson based on GeoGebra?

In this part I am going to describe a lesson based completely on GeoGebra. The lesson subject was the circumscribed circle of a triangle. I am going to present the aim of the parts of the lesson, students' feedback and commitment. In order to make a lesson interesting, certain criteria have to be fulfilled.

2.1. Stating a problem in a stimulating way – modeling and mathematization phase.

An important element of a lesson is a stimulating way in which students are introduced into the lesson subject. The best way is to begin with stating an interesting problem, preferably not a typically mathematic one but one that refers to an everyday situation. I began my lesson with the following sentence: "Three rivers surround a piece of land. Rivers often overflow, flooding the nearby land. Where is the safest place in our piece of land to build a house." The first step of finding the answer is a discussion on what "the safest place to build a house" means. Brainstorming can be used here. Stating a situation that can happen in everyday life helps weaker students to overcome the fear of mathematics and join actively in the discussion. The teacher, through asking leading questions, should direct students to a place equally distant from the three rivers.

At this point you go to the next step, which is creating a model of the problem. Together with the students we drew the following draft, which was to be the presentation of the problem in question.

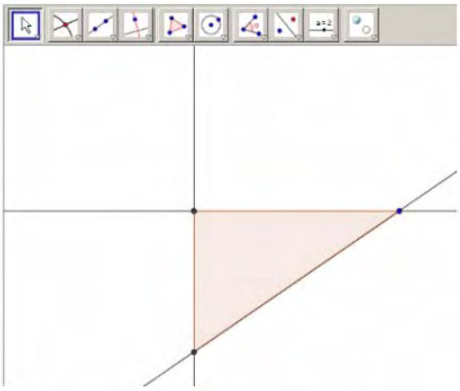

Fig. 1 *The diagram of the problem drawn together with the students with GeoGebra.*

2.2. Theory and Practice

After solving the problem theoretically and creating the situation model, students should put their ideas into practice. What can be suggested here is a virtual experiment of placing a point at an equal distance from three lines.

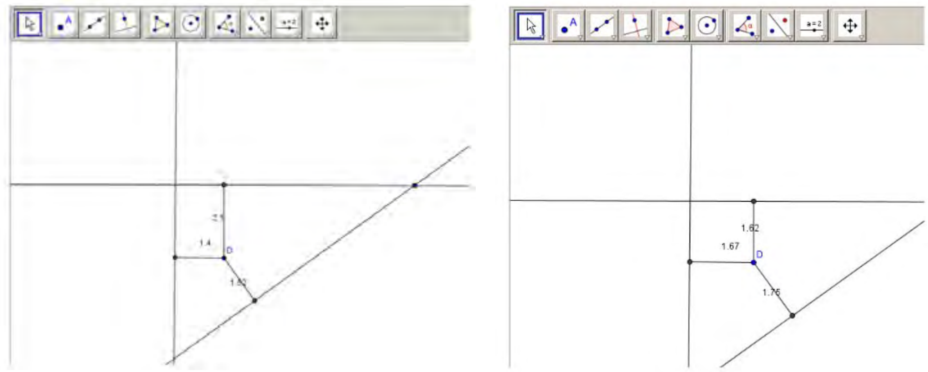

Fig. 2 *The results of the conducted experiment of placing the point at an equal distance from three lines.*

2.3. The reason for introducing new notions.

A great number of mathematic notions seem to students unnecessary since their immediate usage cannot be perceived by them. This can be the case of circumscribed circle of a triangle or other polygons. The next experiment allows finding in an easy way the approximate place for the house, which allows introducing the notion of the circumscribed circle of a triangle as well as explaining its practical use.

Students become convinced that though the solution to the problem is theoretically simple, in practice is impossible to carry out without some extra tools. This incites further thinking and searching for a solution and a new tool. At this point a question as to what figure could help to establish the point can be asked. My students quickly pointed out at a circle and, carrying out the next experiment, used it to mark the place that was equally distant from the three lines.

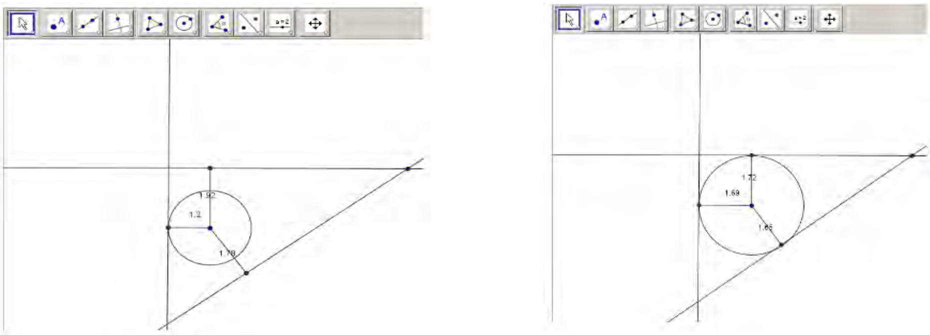

Fig. 3 *Marking the centre point by means of a circle.*

In this way the students introduced the notion themselves. On the basis of the drawing they determined the basic features of such a circle. On the other hand, referring to their school-book, they learnt the name of the new object.

The aim of the next experiment was to find a way to establish a precise centre point equally distant from the three lines. In order to do that, I asked my students to find a point equally distant from two lines only.

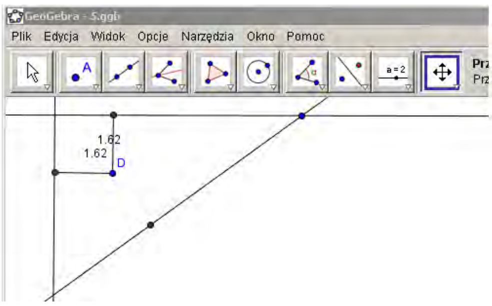

Fig. 4 *Marking the point that partially fulfills the assumptions.*

As a result of the experiment, the students fairly quickly arrived at the right point. Moreover, they noticed that there are many more such points. In the next experiment, the other points of the same characteristic were found.

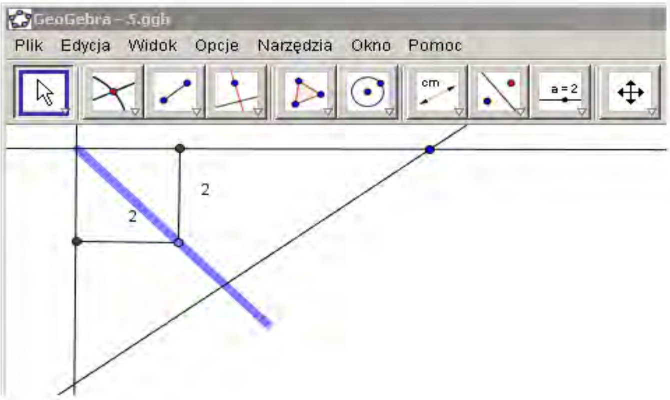

Fig. 5 *The application of the option "a trace of a point" to discover the application of an angle bisector.*

To do this I used showing the option 'a trace of a point'. It should be noted here that the function evoked a great deal of curiosity on the part of the students, and, additionally, let students perceive the already known object, i.e. angle bisector. Previously, angle bisector was used only to divide angles. This time, the students saw its other practical use. This observation was the key element in solving the problem of finding the place to build the house. The students repeated the experiment for each pair of lines and arrived at the sought point.

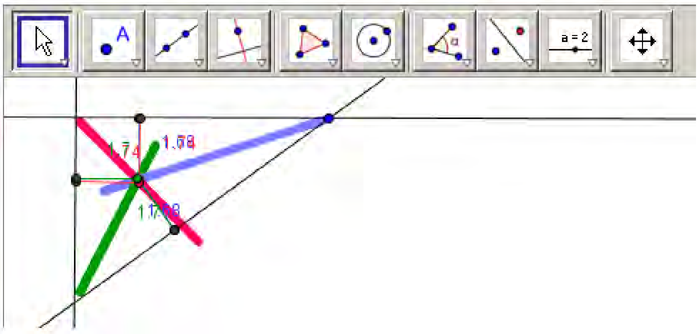

Fig. 6. *The result of the repeated experiment for each pair of the lines.*

2.4. Forming conclusions, hypotheses and interrelations by means of observation

Generalizing, noticing regularities and forming hypotheses and theorems are very important in teaching mathematics. They are characteristic of the 'from the particular to the general' (from an example to a theorem) method. Let me illustrate this by the following example of an experiment which consists of moving the tops of a triangle and observing point D, i.e. the point of the crossing of the bisectors.

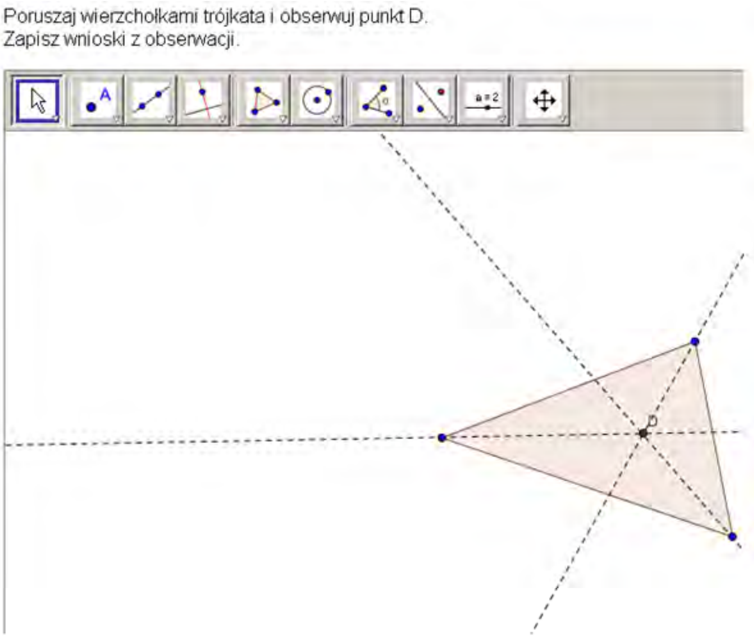

Fig.7 *The application of the experiments for conclusions forming.*

On the basis of the experiment process observation, students formed the hypothesis which says that bisectors of all the sides of a triangle always bisect at one point. They found the confirmation of this hypothesis in the form of a theorem in their schoolbook. Moreover, they formed two conclusions: firstly, those two bisectors are to enough to construct circle center. Secondly, the choice of the bisectors has no significance.

Obviously, this type of experiments may be done on paper in a copy book, but it would require a lot of drawings, on the basis of which conclusions would be drawn. The advantage of the above experiment lies in its dynamic as opposed to the static character of any drawings. In my opinion, it would be worth suggesting an experiment consisting of such a placement of vertexes that the bisectors could not bisect. Of course, it is unfeasible and leads to the conclusion that the bisectors of a triangle bisect at one point.

The above-presented lesson is entirely based on GeoGebra. In practice, the software can be used to construct an experiment which could be a part of a lesson only. In my teaching, I usually use a single experiment to enable students to discover characteristic features, interdependences as well as to formulate theorems. It could be using on different educational levels. The following list consists of the theorems that the students arrived at by means of experiments and observations:

- The theorem of the sum of the measures of internal angles of a triangle;
- Theorems of inscribed and central angles;
- Theorems of circumscribed and inscribed circles of a quadrilaterals;
- Pythagorean Theorem;
- Intercept theorem (Thales Theorem);
- Properties of tangents;
- Properties of geometrical transformations;
- Properties of line bisectors and angle bisectors;
- Properties of polygons.

Experiments created in GeoGebra can be also used in detecting and establishing certain interdependences. An excellent example of this is investigating the influence of the coefficients of a line function on the position of its graph (with younger students) or a similar task but with a square function (for older students).

3. Verifying hypotheses

3.1. The role of examples

Another equally important factor here, apart from discovering characteristics and theorems, is the ability of critical thinking and verifying conclusions and hypotheses. Doing a lot of experiments in the lesson might bring about certain danger. For a great number of students the instruction “prove that…” is identical with the process of verifying on the basis of an example. Therefore, while conducting experiments in the lesson it should be stressed each time that the observations made by students serve only to formulate conclusions and hypotheses. It is important to stress the fact that they need further confirmation. For older students this should motivate them to engage in the verification process. Younger ones, on the other hand, should seek the confirmation of their hypotheses in the school books.

3.2. The role of counterexamples

As it has already been mentioned, experiments cannot substitute a proof, because we are never able to verify experimentally all the possibilities. Yet, one example may suffice to overturn a hypothesis. His should be stressed and taken advantage of during a lesson. Pythagorean Theorem may serve here as a good example. I asked my students a question: „Is the theorem of the rectangular triangle significant?” In other words, is it true the sum of the areas of the squares built on the shorter side of the triangle equals the area of the square built on the hypotenuse? The students did the following experiment.

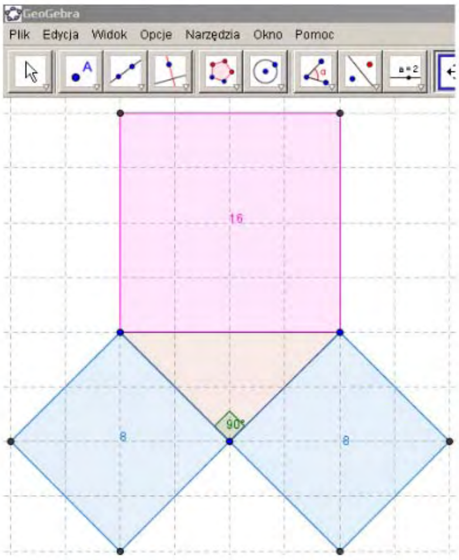

Fig. 8 *The starting point for investigating the necessity of Pythagorean theorem.*

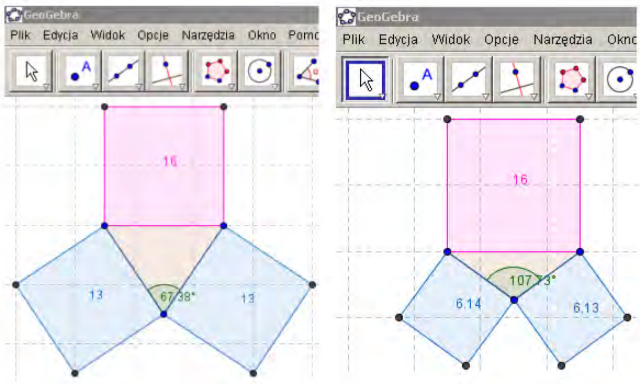

Fig. 9 *The application of experiments to construct counterexamples.*

The students by changing the right-angle triangle into other angles, quickly overturned my hypothesis and claimed that the change of the angle causes the change of the areas of two squares only and that the third one remains unchanged. This conclusion leads to formulating a converse theorem to Pythagorean Theorem.

4. Looking back

G. Polya points out four stages of solving a problem. The last one is checking whether the solution fulfills the assumptions of the task. As far as tasks conducive to do equation or simultaneous equations are concerned it proves very easy, but in the case of geometrical problems it is not necessarily so. I am going to give examples of tasks where verification is very difficult or even, at a certain level of education, impossible without conducting appropriate experiment.

4.1. A house and a tree

"There is a 12 meter-tall tree, 7 meters away from the house. Two thirds of it got broken by the wind. Did the falling tree destroy the house?"

Students made a drawing. They described the given segments and marked the unknown segment "a".

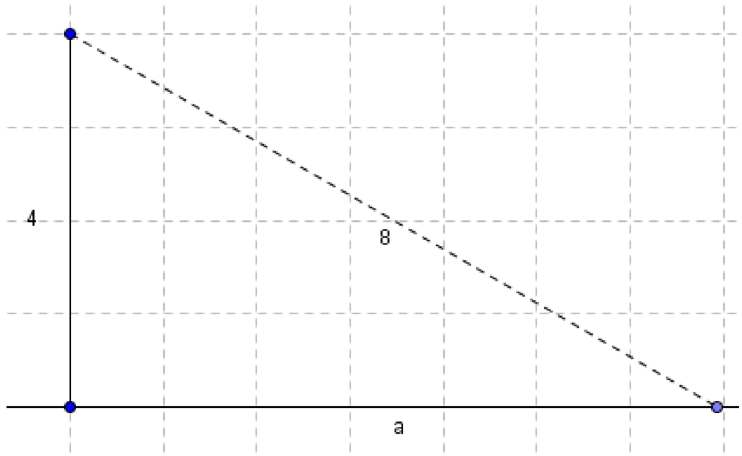

Fig. 10 *The static model of the situation of the task "A house and a tree".*

Using the Pythagorean theorem they worked out the length of the segment "a", which was shorter than 7 meters. On that basis they arrived at the conclusion that the house was not destroyed. In order to prove it, I asked my students to conduct a virtual experiment.

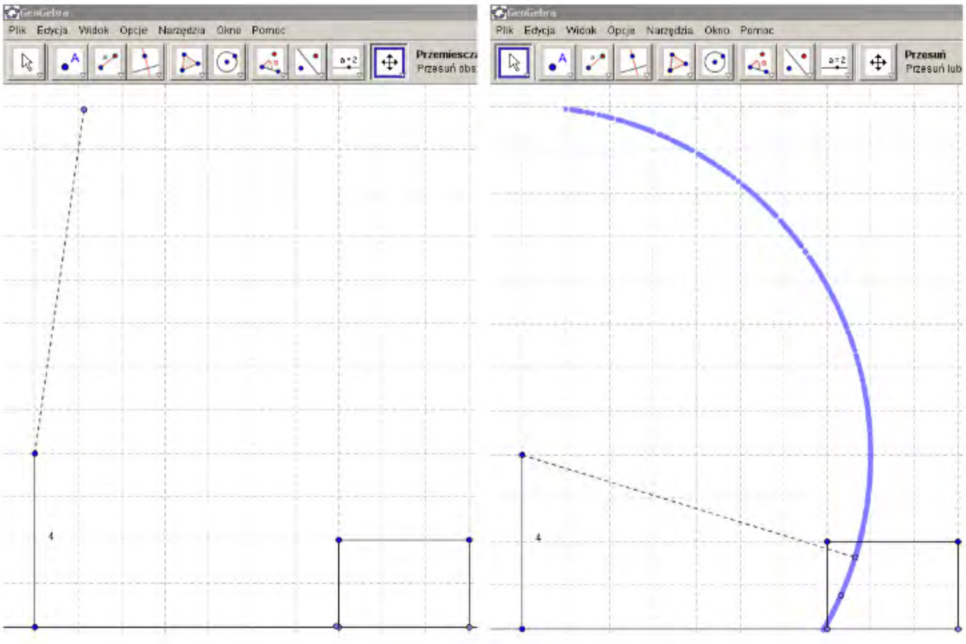

Fig. 10 *The experiment (dynamic model) for the task "A house and a tree".*

The experiment proved that the house had been destroyed. It started a discussion and a question why our calculations were wrong was asked. The students checked their calculations once again. It turned out that they were correct – the mistake was due to the lack of information about the height of the house. That made students try to solve another problem, namely what height of the house would have prevented the house from being destroyed. Although Polya mentions only four steps of dealing with tasks, other sources often quote the fifth one, so-called "looking ahead". It is a phase where students are provoked to put forward new questions or problems on the basis of the previous task. Here was such a phase.

5. Summing-up

The experiments presented above are easy to conduct by any student and allow activating the poorest ones - in fact, those are the ones that should be asked to do the experiments. The average students can do the observations, whereas the brilliant ones can formulate conclusions, hypotheses and theorems. As the experiment cannot be misconducted, poorer students experience the sense of achievement through just conducting it successfully. The important thing for them is to get the feedback from the teacher in the form of praise in front of the class. On the other hand, the brilliant students, formulating conclusions may differ in opinions, which can be used in a discussion. They might even be given an extra task consisting of proving a theorem or overturning hypothesis through searching for counterexample. Virtual experiments, in a simple way, allow for catering for all the kinds of motosensoric preferences (visual, audio and kinesthetic). Student learn through acting (doing an experiment), through watching the results and through formulating and uttering conclusions.

The role of virtual experiments can be summed up with one maxim:

„Tell me and I'll forget, show me and I'll remember, let me act and I'll understand."

Confucius

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