



INFORMATIVE TECHNOLOGY AND THE TEACHING OF MATHEMATICS IN THE TEACHER EVERYDAY WORK AND DIDACTIC INVESTIGATIONS

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Abstract:

The present article makes up the review of chosen scientific accomplishments from the range of studies on using the IT in the mathematical education. He contains the short descriptions of the didactic experiments of the scientific workers of high schools and the mathematics teachers of lower and higher secondary schools which apply IT in his everyday work.

Keywords: graphic calculator; mathematics software; teaching of the algebra; teaching of the geometry; teaching of the analysis

1. Introduction

The contemporary man functions in the social situation determined by the presence of technology. Electronic payment cards find more and more large team of recipients, non-cash internet operations make possible to people making purchases without the need of going out from the house, the pupils of elementary schools are already the owners of mobile phones, and interactive toys make pleasant the children live.

Those transformations bring about new challenges for the teacher. The new tasks standing up in face of the teacher arise from the necessity to prepare the young man to the functioning in the world in which the access to information already has the unrestricted character and the majority of human workings undergo automation. The teacher work has to undergo the gradual modification, leading, in fact, to the change of the way of the education of the youth. This fact seems inevitable. The traditional chalk and board and serving style of the teaching concurring them may not any more suffice for the potential pupil who brought up in the pictorial culture awaits continually new technological challenges.

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2. The activity of The Working group “Mathematics and Computers”

The modern teaching of mathematics is possible among others the thanks of the passion, the commitment and the teachers’ creative attitude, which perceive the needs of the changing world. He assembles such madcaps in his rows, The Association of Mathematics Teachers¹. The activity of Working Groups deserves the special attention among the varied forms of the activity of this organization. The Working Group ‘Mathematics and Computers’ <http://www.snm.edu.pl/grupy/mik/new/> acting from 1994 to 2008 under the direction of prof. Henryk Kałol, is one of the oldest. At present, dr Tadeusz Ratusiński is the coordinator of it. There are many textbooks, articles and even scientific works published within the frame of this organisation. The working up of *The Curriculum of the Teaching of Mathematics at lower secondary school with application of graphic calculators and computer* is one of initiatives of The Working Group *Mathematician and Computers*². Dorota Waloszek, Grażyna Kulesza, Katarzyna Parcia, Jan Sygnowski and Witold Pająk are the authors of this curriculum. The textbooks and tasks books for the pupils, the guide for the teacher with the CD-ROM with corresponding calculators and computer software³.

3. Scientific works

Professor Henry Kałol of The Pedagogical University in Cracow⁴, is the precursor of scientific investigations on the application of modern technologies in teaching of mathematics in Poland. He is the promoter of three doctoral dissertations on using TI in the teaching of mathematics. In his academic work he applies the widely understood TI, among others leading the blended-learning system lectures, supported by the e-learning platform. Prof Kałol conducts, realized on the e-learning platform, all-Polish workshops for pupils talented in Maths from elementary schools, lower and higher secondary schools and for pupils preparing themselves for mathematical contest⁵. He was one of the founders of The

¹ It was registered in 1991 (the decision of The Regional Court in Warsaw) and in the same year on The First National Conference in Bielsko-Biala was chosen The Board of SNM, The Chairman and the Presidium.

² also confirmed by MEN, No. DKW-4014-81/01

³ The both curriculum are accessible on the website: www.dlaszkoly.pl

⁴ Prof. Henry Kałol published 35 textbooks and books, 41 works and scientific character articles and 63 implementing and popularizing character works. He is the co-author of the project Blue Mathematics and New Blue Mathematics (curriculum, textbooks, the exercise books of practices, guidelines for teachers), he directed the team which worked out two projects: *the Teaching of mathematics with the elements of computer science at high school* (curriculum, textbooks, guidelines for teachers, the tasks books and also computer software) and *the Teaching of mathematics at high school with utilization of graphic calculators and computer* (curriculum, textbooks, guidelines for teachers, the gatherings of tasks, calculator software).

⁵ These wheels are led using the e-learning platform (<http://e-dlaszkoly.pl/>).



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Association of Teachers of Mathematics⁶. The scientific seminar for young staff: *The IT in teaching and learning mathematics* is one of the most significant Prof. Kąkol's initiatives. The numbers of scientific studies, including two doctors' works relating to using of graphic calculators, there are the results of the work of participant of this all-Polish seminars.

Edyta Juskowiak is the first of authors of those doctors' work⁷. Exploration and description of the ways of using the graphic calculator by pupils in the process of the teaching and learning mathematics was the aim of her researching. Those researching consisted of the observation of lessons led with the utilization of graphic calculators and the observation of individual pupils work with utilization of the graphic calculator during outclasses occupations. The unique calculator software⁸ AkyRecd, made up the main investigative tool. This software enables the recording of the work done on the graphic calculator (other ones investigative tools were: the sets of tasks, tests, questionnaires and audio recordings).

During their both autonomous and steered by the teacher work, pupils used the graphic calculator for the visualization, doing a great number of test; the concentrations of the attention on the solved task, the investigation of concepts using various options of the calculator, various data representing, the verification of hypotheses. In their independent work pupils used calculators additionally to: experimenting, calculating, the extension of the possibility of the solution of the task, the searching of ideas of the solution of the task, checking the correctness of the result, finding special cases, prolonging the tasks.

My PH thesis is the next one relating to applying the graphic calculator. The aim of it was to exam and describes the influence of constant applying of the graphic calculator in the process of the teaching and learning mathematics on developing of the chosen mathematical activities at 13-16 years old children. The problems dealt with in the work were considered in three investigative areas in the area of the empirical inference, algorithming and work with the mathematical text.

My present research work focuses on the questions relating to the influence of the systematic application of graphic calculators in the teaching of mathematics on the work with mathematical information activity area in higher secondary school. The basic aim of my research is the observation of the influence of permanently applying electronic task cards (table 3) on the efficiency of the work with the mathematical text.

⁶ He was the Secretary of the Board of SNM and directed the works of The Association Teachers Mathematics by 10 years. From 1994 he was the co-ordinator of The Working Group "Mathematician and Computers" and from 1999 the Chief Editor of periodical and the 'The Mathematicians and Computers'. From 1977 he is a member of The Polish Mathematical Society (PTM), in years 2002-2005 he do one's duty Chairman of The Mathematics Didactics Committee of PTM. Since 2006 he is a Chairman of the Forum of Specialists of Mathematical Didactic informal organization assembling the teachers of mathematics, the mathematical didactic specialists and all persons who have at heart problems connected with the teaching of mathematics. He was chosen on The Chairman Association Teachers Mathematics in February 2007.

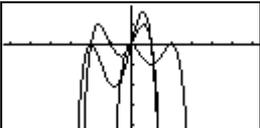
⁷ The scientific worker of The Adam Mickiewicz University in Poznan Department of Mathematics and Computer Sciences.

⁸ The author expressed the agreement on utilization of the software in resources.



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Table 1

The name of the card	The description	
„WYKRWIEL”	<p>This card was designed as the introduction to the subject: „The Bezout theorem”. The using of this card should conduct pupils to autonomous formulating of the contents of the theorem.</p>	<div style="display: flex; flex-direction: column;"> <div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;"> <p>====WYKRWIEL====</p> <p>Temat: WYKRES WIELOMIANU</p> <p>FILE STRIP TEXT CHAR [A+B] D</p> </div> <div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;">  </div> <div style="border: 1px solid black; padding: 5px;"> <p>Poniżej przedstawione zostały wykresy kilku wielomianów trzeciego oraz czwartego stopnia.</p> <p>FILE STRIP TEXT CHAR [A+B] D</p> </div> <div style="border: 1px solid black; padding: 5px;"> <p>Korzystając z dostępnych opcji kalkulatora, przeanalizuj wykresy tych funkcji i sformułuj wnioski uzupełniając</p> <p>FILE STRIP TEXT CHAR [A+B] D</p> </div> </div>
„NIERWIEL”	<p>This card was designed as the introduction to the subject: „Polynomial inequalities”. In the way of the steered observation pupil gets to know the method of the graphic solving polynomial inequalities.</p>	<div style="display: flex; flex-direction: column;"> <div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;"> <p>====NIERWIEL====</p> <p>TEMAT: Nierówności wielomianowe</p> <p>FILE STRIP TEXT CHAR [A+B] D</p> </div> <div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;"> <p>Rozwiń nierówność: $-(x+2)^2(x-3)(x-1)^2 > 0$ $x \in \dots\dots\dots$</p> </div> <div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;"> <p> $-(x+2)^2(x-3)(x-1)^2 < 0$ $x \in \dots\dots\dots$</p> </div> <div style="border: 1px solid black; padding: 5px;"> <p>Korzystając z wykresu wielomianu: $W(x) = -(x+2)^2(x-3)(x-1)^2$</p> <p>WYKRES GRPHE</p> <p>FILE STRIP TEXT CHAR [A+B] D</p> </div> </div>

The next doctoral thesis from the range of the use TI in the teaching is Ph. Dr Magdalena Adamczak’s dissertation⁹ relating to the application of new technologies in the teaching of mathematics on the academic level. The possibility of the utilization of new technologies in the process of the teaching of mathematics on the academic level with the exemplification in ordinary differential equations was the main aim of this dissertation.

The computer (more exactly: *Derive* software from the range of so-called computer algebra, *WinPlot* graphic software and *PowerPoint* – to creating the presentation) was used on classes on ordinary differential equations¹⁰. The diverse forms of the work were applied on occupations with students: from the introduction for the whole group, through the work in peers for students’ individual work. The question of the field of directions was worked

⁹ The research worker of Adam Mickiewicz University in Poznań, Department of Mathematics and Computer Sciences.

¹⁰ During the realization of following subjects: “The definition of the ordinary differential equation. Preliminary notions”, “The geometrical interpretation of the figure differential equation”, “Clairaut equation differential”, “Peculiar solutions. Theorems on an existence and the unequivocal of solutions”, “Equation the differential families of curves. Orthogonal curves”, “The definition of the differential equation of n-degree. Basic notions”, “The homogeneous simultaneous linear differential equations with constant coefficients”.



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out wide although this subject was not so far in the programme of practices. The interesting fact is the experience captured during occupations relating to the field of directions became inspiration to different than the traditional way solving of tasks relating to certain questions from differential equations. The work the students, which decided to mark the graphic solution peculiar equations – in the different way than got to know on practices – using the notion of the field of the directions which it appeared on one of first occupations, was the interesting example on this. Students on certain stage of solving the given equation $y = xy' + (y')^2$ received the equation in the following figure $y' = -x/2$, which they did not solve farther, but they crossed out the field of the directions of this equation. Then in the same dialogue window of the *WinPlot* software they made the graph of the general solution (in fact several special solutions – Fig.1). Using the tangent property of the curve of the singular solution (discovered during classes) they presented, without making calculations, which parabola would be the curve of the singular solution and they guessed its equation.

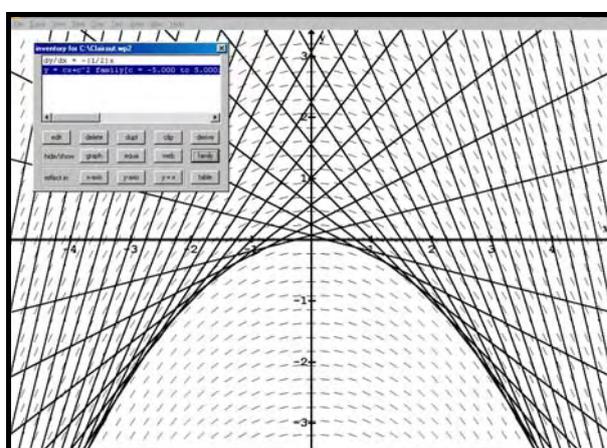


Fig. 1 The graph of the solutions of the Clairaut equation.

Dr Witold Pająk¹¹ conducted research¹² on the role of CABRI software in assimilating mathematical contents by the pupils of the lower-high school. Two kinds of empirical research decisive about the whole structure of the research conduct were foreseen in the work.

To examine whether the CABRI software can help in weakening the influence of the principal directions in the process of reception of such a drawing by the pupil, the

¹¹ The vice director of upper secondary school in Oświęcim (Powiatowy Zespół Szkół im. S. Konarskiego w Oświęcimiu).

¹² The title of the doctor's thesis: "The research of the role of CABRI software in assimilating mathematical contents by the pupils of lower-secondary school".



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individual test was planned 1 – the height of the triangle. The script of the investigation is as follows. The definition of the height of the triangle was reminded in the preliminary conversation with the pupil (it could take a descriptive form or functional formulation in the form of the enumeration of the next stages of drawing, together with the hand-made draft); reciting „the bookish” phrase was not the point. The pupil in the research was observing static drawings 2a) and 2b) illustrating triangles of proportionally equal sides, on the computer screen. He answered the following question in each of the cases: *Is a segment the height of the triangle every time?* As usual, the pupils’ answers could be more or less positive. The pupil marked the degree of their assertion (certainty) on the scale which 0 and 1 were extreme values.

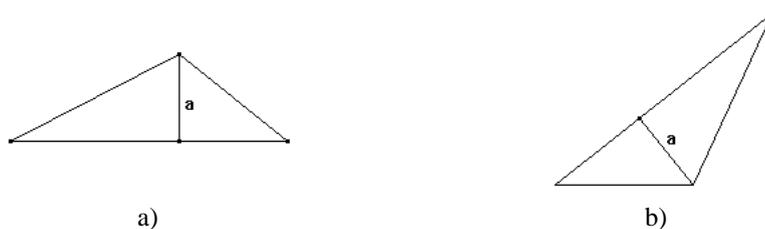


Fig. 2 Static drawings illustrating triangles of proportionally equal sides.

The pupil’s independent answers were noted down. Next he was encouraged to observe on the screen of the monitor the transformation which moves the triangle from the primary position 2.a to the final position 2.b. Firstly, the translation to the position in the figure 3.a was carried out, then the rotation around one of the triangle topics to conduct to the final position (Fig. 3.b). The situation when the pupil wants to do the dislocation of the triangle on the screen itself, is the best one in this type of the test; the pupil can turn it round and repeat several times.

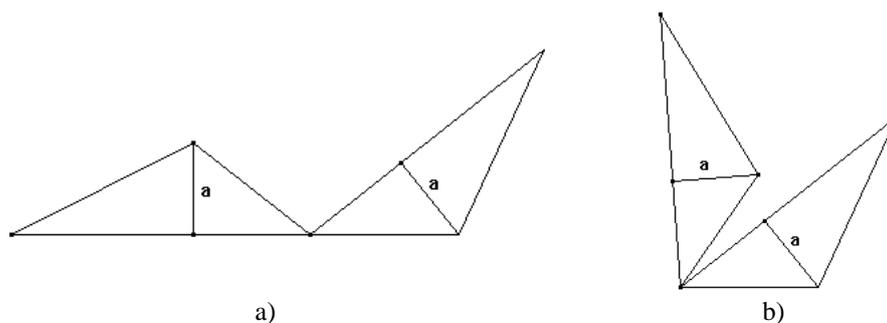


Fig. 3 Translation and rotation of the static drawings illustrating triangles of proportionally equal sides.



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The question formed in p. 2) was renewed in the successive phases of transformation (in the peculiarity after putting on the triangles); the answer certainty degree on the subject of the height of the triangle founding in the final position were expected also. In this part – using the movement in CABRI – participated only those pupils who associated with the drawing 2b) incorrect answers in the static part of the investigation or which marked the degree of the certainty smaller than 1.

In the case of any change of the pupil's attitude in the course of the experiment carried out on the screen, the test was renewed - after about 3 months - asking the question set in p. 2). Let's name this stage the renewed research.

The results of conducted resources demonstrate that CABRI and the traditional way of drawing were not competitive systems (or alternative) but mutually complementary in the process of the teaching. Using the CABRI software during the realization of the curriculum (regarding mathematical contents of the teaching), it should be united with the exchange of opinions on the subject of made observations and their mathematical interpretation among pupils or a pupil and a teacher. The common dialogue seems to be the indispensable factor in the correct analysis of the mathematical situation, and also in proper understanding of the role of the didactical means by pupils.

There are other doctoral dissertations on using of the technology in teaching. Katarzyna Wadoń-Kasprzak¹³ leads resources on formation of the notion of the parameter among the pupils of lower-secondary school and also on the understanding of this notion by them. The aim of those investigations is to find of the solution of the following investigative problem: Can the computer software be useful in the process of formation of the notion of the parameter in all meanings and in what degree? The files of the computer software created by pupils during solving tasks with the TI InterActive software were essential investigative tools in the above mentioned investigations.

Let's consider the f function given the formula: $f(x) = \frac{x^2}{c} - 7$, where $c \neq 0$.

- a) What is the influence of the value of the c parameter on the shape of the parable?***
- b) For what parameter 'c' value the distance between zeros of the function is equal 6?***

To solve the above mentioned task on the lesson Gosia - pupil of the class III b used the computer software TI InterActive!. She prepared the following drawing (Fig. 4), changed the value of the "c" parameter and observed the change of the position of the parable in the coordinate system.

Renata Wojtuś prepares the next doctoral dissertation of the use of the technology in the teaching¹⁴. The subject of her investigations is connected with application by the pupils'

¹³ The mathematics teacher in lower-secondary school in Kęty (Zespół Szkolno-Gimnazjalny nr 2 w Kętach).

¹⁴ The high school in Kielce (Wyższa Szkoła Umiejętności im. S. Staszica w Kielcach)



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mathematical computer software in the process of the teaching and learning mathematics, and in particular, in the autonomous out-of-classroom pupil's work.

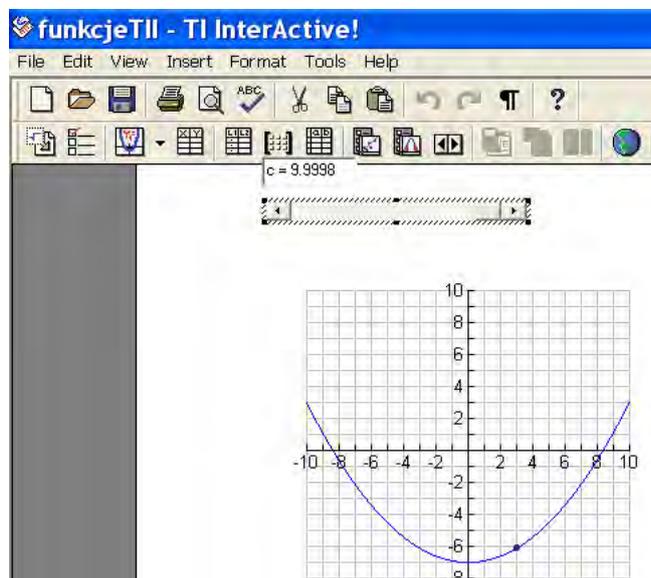


Fig. 4 Pupil's drawing designed with TI InterActive software

The example of the application of the mathematical computer software (Winplot) in the out-of-classroom pupil's work is presented below. It is connected with the square function (this task was from the long-term homework). The pupil solved the task:

Examine the changes of the position of the graph of the function given by formula $y = ax^2 + bx + c$, $a \neq 0$ in dependence from coefficients: a, b, c .

To solve this task the pupil used the software Winplot. She experimented with the various values of coefficients a, b, c . The software prepared for her the graphs of the square function quickly and correctly and she could observe appearing regularities of those graphs. Dynamic and smooth (almost continuous) changes of one of the parameters a, b, c led to the changes of the position of the vertex of the parabola described by the equation $y = ax^2 + bx + c$. The figure 5 is the „screenshot” of the software trough experimenting with the coefficient a . The pupil discovered that with changing the a coefficient, the vertex of the



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parabola lies on line described by the following equation $y = \frac{1}{2}bx + c$. Using the Winplot software she verified her observation.

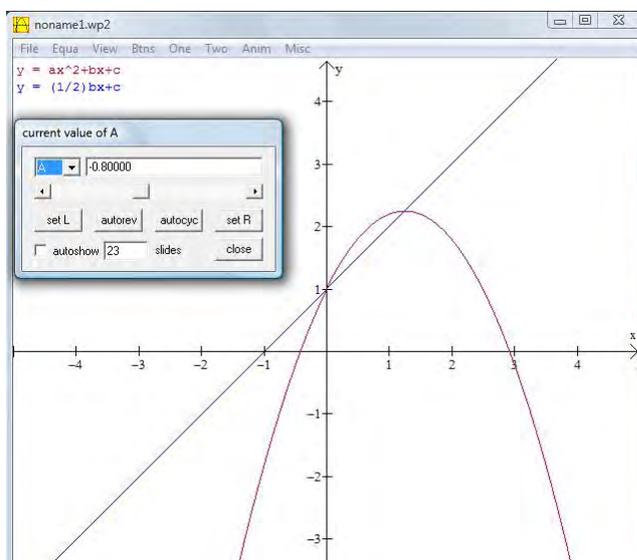


Fig. 5 Screenshot of the software trough experimenting with the coefficient *a*.

Experiments with the concrete value of coefficients *a*, *b* and *c* brought her the following interesting conclusions:

- if the values of parameters *b* and *c* are settled randomly, then the set of the vertex of the family of the parabolas described by the equation $y = ax^2 + bx + c$, where $a \neq 0$, creates line of the form $y = \frac{bx}{2} + c$.
- if the value of parameters *a* and *c* are settled randomly, then the set of the vertex of the family of the parabolas described by the equation $y = ax^2 + bx + c$, where $a \neq 0$, he creates the parable of the form $y = -ax^2 + c$.
- if the value of parameters *a* and *b* are settled randomly, then the set of the wertext of the family of the parabolas of which everyone is described by the equation $y = ax^2 + bx + c$, where $a \neq 0$, he creates straight line of the form $y = -\frac{b}{2a}$.

Another doctoral dissertation is being prepared by Janina Duda¹⁵. The main aim of her experiment is to exam whether the graphic calculator can provoke the active attitude of the pupil who has mathematical predispositions in the following range: experiments, the perception of the problems, formulating and verification of the hypotheses.

¹⁵ The mathematics teacher of the upper secondary school (Zespół Szkół Ogólnokształcących w Zabrze).



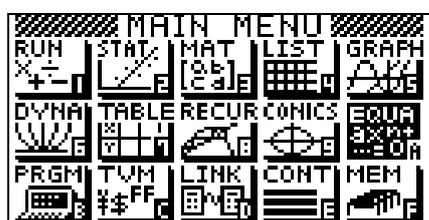
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The task: Solve using the graphic calculator the following arrangements of equations:

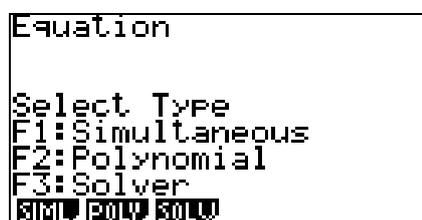
$$\begin{array}{l}
 \text{a)} \begin{cases} 18x + 19y = 20 \\ 21x + 22y = 23 \end{cases} \quad
 \text{b)} \begin{cases} 1252x + 1253y = 1254 \\ 1255x + 1256y = 1257 \end{cases} \quad
 \text{c)} \begin{cases} -24x - 23y = -22 \\ -21x - 20y = -19 \end{cases}
 \end{array}$$

What did you notice? Formulate the suitable conclusion and try him to justify it. Try to build the different simultaneous of equations of certain special properties.

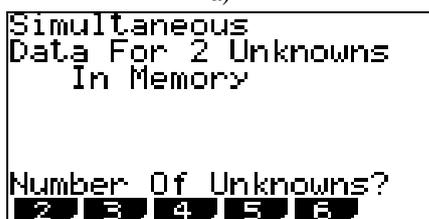
All pupils solved the simultaneous equations using the graphic calculator, using mode of work „EQUA” (Equation) and option „SIML” (Simultaneous) (Fig. 6.a, 6.b, 6.c). Pressing [F1](2) they got table (Fig. 6.d) After inscription of the next coefficients of the arrangement to this table (Fig. 6.e) and pressing the button [F1] (SOLV) got solution of simultaneous equations (Fig. 6.f).



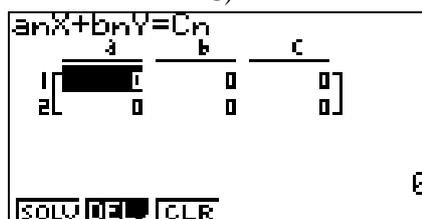
a)



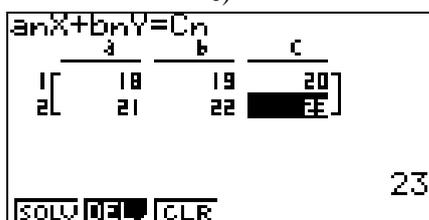
b)



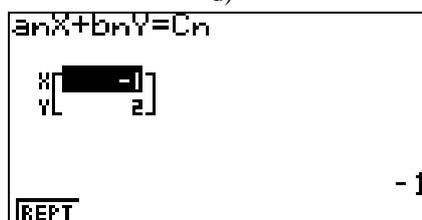
c)



d)



e)



f)

Fig. 6 Solving the simultaneous equations using the graphic calculator using mode “EQUA” and option “SIML”.



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After solving three given simultaneous equations every one of them had the same solution: $x = -1$ and $y = 2$, all pupils noticed that the difference between the coefficients equals 1 and they wrote down the suitable hypothesis (conclusion). After that they “went their own way”: pupils invented various simultaneous equations, found solutions for many cases, formulated hypotheses and verified them, empirically, and four pupils also theoretically. Using symbolic records they wrote down at least two examples of their investigated arrangements on the card of the work and wrote down their conclusions.

4. Conclusion

The review of didactic proposals, relating to the using of the IT in teaching of mathematics included in the present article, shows significant role of this technology in the formation of mathematical concepts and developing mathematical activities. The total of experiments realized in this range shows the necessity of engaging computer tools into the process of teaching, especially in the context of curriculum changes happening currently in Poland. Polish teachers’ and academics’ experience in this range is huge. Modern education postulated in the assumption of the educational reform has the chance to be realized on condition that experience sharing as well as the exchange of results of the scientific investigations takes place. They should carry the value of applicability, essential for the improvement of teaching efficiency.

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